Department of Mathematics & Philosophy of Engineering

Faculty of Engineering Technology

The Open University of Sri Lanka

Course: MPZ 3132 Engineering Mathematics IB **Assignment No.04 Academic Year – 2011/2012**

Instructions

- Answer all questions
- Write your address back of your answer scripts
- Use both sides of paper when you are doing assignment**.**
- Please send the answer scripts of your assignment **on or before the due date** to the following address.

Course Coordinator – MPZ 3132

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You can collect model answers from virtual class ***(www.ou.ac.lk)***

User name - student0 Password – MPZ3132

1. State the parallel axis theorem and perpendicular axis theorem.

1.1. Find the moments of inertia of a uniform circular hoop of radius r and mass M.

1.1.1. A lamina in the form of annulus mass M has the inner and outer radii$a$ and b respectively. Prove that the moment of inertia of the above lamina is $\frac{1}{2}M(a^2+b^2)$.

1.1.2. The above lamina can rotate freely about a fixed axis which is tangent to the outer boundary of the lamina. Show that the period of small oscillations of the lamina is $\pi\sqrt{\frac{a^2+5b^2}{gb}}$.

2. Find the moments of inertia of a uniform rod length $2a$ and mass M about an axis which is perpendicular the rod and passing through one end.

2.1. A uniform rod AB of mass M and length $2a$ is free to smoothly turn in vertical plane about a horizontal through A. A particle of mass m is fixed to the rod at B.

2.1.1. Find the moments of inertia of the system about the axis which is perpendicular to the rod through A.

2.1.2. When the rod is hanging in equilibrium, it is set to move with an angular velocity ω. Find the range of ω for which the particle describes complete circles about A.

3. A body moving so that, at time t, its is mass is m, its velocity is $\boldsymbol{v}$ and the resultant force acting on the body is $\boldsymbol{F}$. Suppose that at this instant, a particle of mass δm and velocity $\boldsymbol{u}$ coalesces with the body so that, at time δt, the body has the mass $m + \delta m$ and velocity $v + \delta v$. Prove that $$\boldsymbol{F} = m\frac{d\boldsymbol{v}}{dt} + (\boldsymbol{v} - \boldsymbol{u})\frac{dm}{dt}.$$

3.1. A body with initial mass m is projected vertically upwards with speed $\frac{g}{k}$ where k is a constant. At any subsequent time t its speed is v and its mass is me^{kt}. The particles of added mass were all at rest when picked up by the body. Find the mass of the body at its highest point.

3.2. A particle is projected vertically upward with speed u in a resistive medium at point A. The resistive force on the particle is mkv^2 where v the velocity of the particle is and k is a constant. Prove that the maximum height attained by the particle is $\frac{1}{2k} ln\left(1 + \frac{g}{k}u^2\right)$. If the particle will return to the point A with velocity αu prove that $\frac{1}{\alpha^2} = 1 + \frac{ku^2}{g}$.

4. A particle P moves on a plane. At time t the polar coordinates of P is (r, θ) and the position vector of P is $\boldsymbol{r}$. If the unit vector along $\boldsymbol{r}$ is $\boldsymbol{l}$ and the unit vector perpendicular to $\boldsymbol{r}$ and θ increasing direction is $\boldsymbol{m}$

4.1. prove that the velocity $\boldsymbol{v} = \dot{r}\boldsymbol{l} + r\dot{\theta}\boldsymbol{m}$ & the acceleration $\boldsymbol{f} = (\ddot{r} - r\ddot{\theta})\boldsymbol{l} + \frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})\boldsymbol{m}.$

4.2. Particles P and Q of masses m and km respectively are connected by an inextensible weightless string of length $2l$. The string passes through a smooth small fixed ring O on a smooth horizontal plane. At initially the string is taut and $OP = l$. Then P is projected on

the plane with speed u perpendicular to OP. After time t, $OP = r\ (< 2l)$. Prove that the tension of the string is $\frac{kml^2u^2}{(1+k)r^3}$.

5. A lamina is immersed in a homogenous liquid prove that the thrust on the one surface is equal to

$Area\ of\ immersed\ surface \times perssure\ on\ the\ center\ of\ the\ immersed\ surface$

5.1. A uniform rectangular lamina $ABCD$ is immersed vertically in a homogeneous liquid such that AB and CD are horizontal. The distances to AB and CD from the surface of the liquid are h_1 and h_2 respectively. Prove that that the centre of pressure of the lamina is at the distance $\frac{2}{3}\left(\frac{h_1^2+h_1h_2+h_2^2}{h_1+h_2}\right)$ from the surface.

5.1.1. The inside cross section of a closed right cone is a square of side $2a$ and the height of the inside of the cone is h. The cone is completely filled with a homogeneous liquid. The weight of the liquid inside the cone is w . If the cone is kept such that the axis of the cone is horizontal, find the reaction on the angled surfaces of the cone.

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